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SUPPLEMENTS

Statistical techniques developed by the UKDMC to calculate the 90% c.l. upper limit of the WIMP component of the Galactic dark matter.

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This paper outlines statistical techniques developed by the UK Dark Matter Collaboration to convert the resultant energy spectra from NaI crystal scintillation detectors to a 90% confidence level upper limit to the rate of the hypothetical weakly interacting massive dark matter particle, the WIMP. In simple counting systems a chi-square surface minimisation technique is used, the 90% c.l. arising from Monte Carlo simulations. In detectors that exhibit electron from nuclear recoil discrimination a comparison between the observed spectrum and that from gamma and neutron calibrations is performed with chi-square, Kolmogorov-Smirnov and Cramer-von-Mises tests to give the most likely signal, with Monte Carlo simulations again yielding the 90% c.l.

1. GENERAL PRINCIPLES.

Underground experiments involved in the search for the weakly interacting massive particle (WIMP) component of the Galactic dark matter are not yet sensitive enough to directly detect and tag these particles. Results from these detectors are thus presented as upper limits to the WIMP cross section or rate. This paper outlines some of the statistical techniques developed by the UK Dark Matter Collaboration to calculate this 90% confidence level upper limit from the NaI scintillation detectors running in the Boulby mine. All such systems consist, in principle, of a NaI crystal viewed by two photomultiplier tubes in coincidence. The coincidence is used to reduce event triggers arising from photomultiplier noise pulses, with a summed signal from both tubes being used in subsequent analysis to calculate the observed energy spectrum.

There are two categories of NaI experiments currently being employed by the UK Dark Matter Collaboration, and other research groups. Firstly, simple counting detectors in which the background beta and gamma radioactivity of the detector components is reduced to a minimum with the resultant low level energy spectrum being used to calculate the WIMP rate upper limit. This type of detector is incapable of actually identifying WIMPs directly as no differentiation is made between different scintillation events and so all observed pulses must be assumed to be candidate dark matter particles. This type of detector system is known as stage 1. The second category of detectors, building on the work of the first, are discrimination devices where some set of measured parameters, such as pulse time constant or ratio of UV to visible light, is used to characterise the scintillation pulse. Due to the different mechanisms excited by nuclear and electron recoils, induced by a WIMP and background gamma respectively, these signal parameters may be used to distinguish the different incident particles. The gamma background may then be statistically removed from the observed energy spectrum. These second detector systems are known as stage 2.

The general principle used in calculating upper limits to the WIMP dark matter rate is to fit a theoretical curve for a given WIMP mass to the measured energy spectrum. Both curves are plotted in 'differential rate units' as counts/kg/day/keV versus energy. By adjusting the WIMP cross section (σ), or underlying rate (R_0/r), the theoretical curve can be matched to the observed spectrum. The theoretical curve must account for such corrections as the recoil efficiency, the form factor, energy resolution of the detector and Poisson correction due to low number of photoelectrons in a coincident system. The calculation of the 90% c.l. upper limit to the WIMP rate is dependent on the type of experiment.

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2. CALCULATION OF UPPER LIMIT.

2.1 Stage 1 Detectors.

For setting upper limits, every event recorded in these detector systems must be assumed to be due to a dark matter particle. The theoretical WIMP curve is thus fitted to the observed spectrum directly. The problem of calculating the 90% c.l. upper limit is to determine a technique that is robust enough not to depend on a single point which will produce an artificially low result dependent on a downward fluctuation. One such technique, developed by the UK Dark Matter Collaboration is the use of a chi-square minimisation procedure. The theoretical curve and the observed spectrum are compared over a varying range of energy bins and the chi-square value calculated[1,2]:

$$\chi^2 = \sum_i \frac{(N_i - e_i)^2}{N_i} \tag{1}$$

where N_i is the observed number of counts in an energy bin i, with e_i being the expected number. A chi-square surface is created as a function of the dark matter particle rate and the number of bins used in the chi-square function, see figure 1. The minimum of this surface, χ^2_{min}, is used as a robust estimator of the most probable dark matter particle rate. The robustness of this technique has been verified by ensuring that the resultant dark matter rate is insensitive to the removal of random points from the energy spectrum.

The 90% c.l. of the dark matter particle rate can now be determined from the chi-square surface by stepping up from the minimum of the chi-square surface and finding the value of the dark matter particle rate which has a value of chi-square such that

$$\chi^2 = \chi^2_{\min} + \Delta\chi^2$$

where $\Delta\chi^2$ is the chi-square interval required to define an ellipse whose projection onto the R_0/r axis includes 90% of the data. This value may be calculated analytically for Normally distributed data points as $\Delta\chi^2 = 2.71^2$. Alternatively the value of this chi-square interval may be determined by Monte Carlo simulation from the actual data. The data points are repeatedly fluctuated according to their errors and the minimum of the chi-square surface determined in each case. The chi-square interval may then be calculated from the ellipse which contains 90% of the fluctuated data set.

2.2 Stage 2 Detectors.

In these detector systems the data are recorded in terms of both deposited energy and some discriminatory parameter, such as pulse risetime or ratio of ultraviolet to visible light. For each energy slice the background distribution in this discriminatory parameter is then compared against spectra recorded with neutron and gamma calibration sources. This comparison can be done directly against the normalised differential curves, or against normalised integral curves. If neither a dark matter nor a neutron signal is present, and all systematics have been correctly accounted for, then the background curve will fall on the gamma calibration curve, within statistical errors. The analysis technique in this case is to determine the 90% c.l. upper limit to the neutron / dark matter signal that could exist in a given background spectrum. Once this 90% c.l. upper limit of the dark matter signal has been extracted the theoretical dark matter rate curve may be fitted directly to the lowest point of the resultant spectrum.

Three different statistics have been used to test the goodness-of-fit of the background spectrum, N_1, to the calibration spectra, N_2, the chi-square[1,2], the Kolmogorov-Smirnov[1,2] and the Cramer-von-Mises[2] statistic. From Monte Carlo simulations it is found that the chi-square statistic, equation 2, is most sensitive when the background consists of all gamma or all neutron pulses.

$$\chi^2 = \sum_i \frac{(N_1 - N_2)^2}{N_1 + N_2} \tag{2}$$

The Kolmogorov-Smirnov statistic, d in equation 3, which essentially uses the maximum separation, D, of the two cumulative distributions, F_1 and F_2, to calculate the probability that they were drawn from the same parent population is found to be most sensitive for a background combination of half gamma and half neutron pulses.

$$d = \sqrt{\frac{(N_1 N_2)}{(N_1 + N_2)}} D \quad (3)$$

where $D = \max[F_1(x) - F_2(x)]$

Finally the Cramer-von-Mises statistic, C in equation 4, which uses the mean square difference between the two cumulative distributions to calculate the probability that they both arise from the same parent population is found to be equally sensitive over the whole range of neutron fraction.

$$C = \frac{(N_1 N_2)}{(N_1 + N_2)} W^2 \quad (4)$$

where $W^2 = \sum_i [F_1(x_i) - F_2(x_i)]^2$

For a given Monte Carlo run for which a fraction f_n of neutrons is injected the width of the returned distribution of most probable neutron fractions, σ_s , is found to be a consistent figure of merit of the experiment for all statistical tests. If the gamma and neutron calibration curves are approximated to Gaussian curves with a separation of S and standard deviations of σ_γ and σ_n respectively then the figure of merit is given by equation 5, where N is the number of events drawn in the simulation, i.e. the number of background events. Figure 1 shows this figure of merit superimposed upon Monte Carlo results for varying values of σ_γ and σ_n

$$\sigma_s = \frac{(\sigma_n f_n) + (\sigma_\gamma (1 - f_n))}{S\sqrt{N}} \quad (5)$$

To calculate the most probable dark matter signal fraction, f_n, in a background spectrum a composite curve is created from the gamma and neutron calibration curves. The fraction of neutron calibration curve is varied from 0.0 to 1.0, thus effectively utilising the Bayesian statistical approach. The composite curve created is compared to

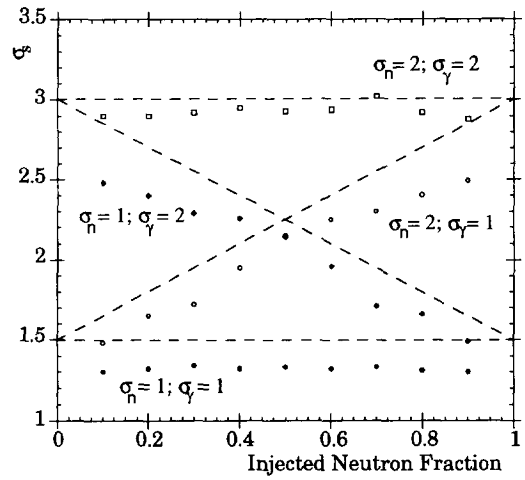

Figure 1: Illustration of figure of merit for varying values of σ_γ and σ_n

the background spectrum using the above statistical tests.

A Monte Carlo simulation is then performed to determine the 90% C.L. upper limit to the neutron fraction, f_{90}. This value is used to scale the background count rate for this particular energy slice, giving the 90% C.L. upper limit to the dark matter signal rate.

The optimum technique for combining the 90% c.l. upper limit signal rates from different energy slices is currently under investigation. Once combined the theoretical dark matter rate curves for each WIMP mass are then directly fitted to the 90% c.l. upper limit rate curves to give the upper limit to the WIMP cross section and underlying rate[3]. A further advantage of this statistical technique of determining the upper limit of the WIMP rate is that spectra from several crystals may be combined to improve the limit, even those with differing energy thresholds and responses, or those from different experimental groups.

3. REFERENCES.

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