Spin factors - revised tables.

Because the full nuclear shell model treatments contain contributions from both p and n, model-independent results corresponding to the simple '$\lambda^2 J(J+1)$' value don't exist: the single-particle

$$I_{s(sp)} = C_{WN}^2 \lambda^2 J(J+1).$$

is replaced by

$$I_s = [C_{Wp} < S_p > + C_{Wn} < S_n >]^2 (J+1)/J,$$

The effective $\lambda^2 J(J+1)$ is therefore:

$$\begin{aligned} \lambda^2 J(J+1)_{eff} &= I_s / C_{WN}^2 \\ &= [< S_p > + (C_{Wn}/C_{Wp}) < S_n >]^2 (J+1)/J \quad \text{(odd proton)} \\ &= [(C_{Wp}/C_{Wn}) < S_p > + < S_n >]^2 (J+1)/J \quad \text{(odd neutron)} \end{aligned}$$

(for details see *e.g.* Jungman, Kamionkowski & Griest [1] – note transposition of S_{01} and S_{11} in their Table 5)

Table 4 of Lewin & Smith [2] only gave values of C_{WN}^2, concealing the signs of the C_{WN}. Also, that table contains a misprint – $\cos^2\theta$ should be $\cos^4\theta$ ($\tilde{B}$ rows) – and a sign error resulted in wrong values for $C_{WN}^2(\tilde{Z})$. The following table is a corrected and expanded version. Note that the usual notation now is a_p ($\equiv C_{Wp}$), a_n ($\equiv C_{Wn}$).

WN	C_{WN} NQM	C_{WN} EMC [3]	C_{WN}^2 NQM	C_{WN}^2 EMC [3]	$\frac{\sigma_{WN}\big\vert_{spin}}{\mu^2 I_s}$	$\frac{\sigma_{WN}\big\vert_{spin}}{\sigma_{\nu_M N}}$
$\tilde{\gamma}p$	0.38 ± 0.01	0.31 ± 0.01	0.14 ± 0.01	0.096 ± 0.009	$\frac{4}{\pi}\left(\frac{e}{m_{\tilde{q}}c}\right)^4$	$\left(\frac{M_F}{m_{\tilde{q}}}\right)^4$
$\tilde{\gamma}n$	-0.04 ± 0.01	-0.11 ± 0.01	0.002 ± 0.001	0.012 ± 0.003		
$\tilde{H}p$	0.63 ± 0.01	0.68 ± 0.03	0.40 ± 0.02	0.46 ± 0.04	$\frac{8G_F^2}{\pi\hbar^4}\cos^2 2\beta$	$4\cos^2 2\beta$
$\tilde{H}n$	-0.63 ± 0.01	-0.58 ± 0.03	0.40 ± 0.02	0.34 ± 0.03		
$\tilde{B}p$	0.39 ± 0.01	0.32 ± 0.02	0.15 ± 0.01	0.10 ± 0.01	$\frac{1}{\pi}\left(\frac{e}{m_{\tilde{q}}c}\right)^4 \frac{1}{\cos^4\theta_W}$	$\left(\frac{M_F}{m_{\tilde{q}}}\right)^4 \frac{1}{4\cos^4\theta_W}$
$\tilde{B}n$	-0.03 ± 0.01	-0.10 ± 0.02	$(7 \pm 5) \times 10^{-4}$	0.010 ± 0.003		
$\tilde{Z}p$	0.68 ± 0.04	0.20 ± 0.08	0.46 ± 0.06	0.04 ± 0.03	$\frac{4}{\pi}\left(\frac{e}{m_{\tilde{q}}c}\right)^4 \tan^4\theta_W$	$\left(\frac{M_F}{m_{\tilde{q}}}\right)^4 \tan^4\theta_W$
$\tilde{Z}n$	1.17 ± 0.04	0.69 ± 0.08	1.4 ± 0.1	0.5 ± 0.1		

Table 4 (revised): Values of WIMP-nucleon spin factors; $M_F = \sqrt{8}\, M_W \sin\theta_W \simeq 109\ \mathrm{GeVc^{-2}}$

With values of Δq from Ellis & Karliner [3] and $<S_p>$, $<S_n>$ from Ressell [4] (F), Ressell & Dean [5] (Na, Te, I, Xe), Engel *et al.* [6] (Al), Ressell *et al.* [7] (Si, Cl), Dimitrov, Engel & Pittel [8] (Ge), and Engel *et al.* [9] (Nb), Table 3 of [2] can be revised and extended:

Isotope	odd nucleon	J	$\lambda^2 J(J+1)$		$\lambda^2 J(J+1)_{eff}$				comments
			single particle	odd group	$\tilde{\gamma}$	$\tilde{H}$	$\tilde{B}$	$\tilde{Z}$	
^{1}H	p	1/2	0.75	0.75	0.75	0.75	0.75	0.75	
^{13}C	n	1/2	0.083	0.101					
^{19}F	p	1/2	0.75	0.647	0.686	0.698	0.685	0.593	
^{23}Na	p	3/2	0.15	0.041	0.097	0.089	0.097	0.167	
^{27}Al	p	5/2	0.35	0.087	0.155	0.141	0.156	0.278	
^{29}Si	n	1/2	0.75	0.063	0.055	0.053	0.056	0.050	
^{35}Cl	p	3/2	0.15	0.077	0.0051	0.0041	0.0051	0.016	
^{43}Ca	n	7/2	0.321	0.152					
^{73}Ge	n	9/2	0.306	0.065	0.105	0.144	0.099	0.183	
^{93}Nb	p	9/2	0.306	0.162	0.228	0.188	0.231	0.666	
^{125}Te	n	1/2	0.75	0.161	0.242	0.245	0.242	0.248	'Bonn A'
					0.315	0.314	0.315	0.313	'Nijmegen II'
^{127}I	p	5/2	0.35	0.007	0.112	0.084	0.114	0.455	'Bonn A'
					0.154	0.126	0.156	0.466	'Nijmegen II'
^{129}Xe	n	1/2	0.75	0.124	0.235	0.319	0.221	0.404	'Bonn A'
					0.209	0.244	0.203	0.277	'Nijmegen II'
^{131}Xe	n	3/2	0.15	0.054	0.068	0.078	0.066	0.088	'Bonn A'
					0.056	0.069	0.054	0.081	'Nijmegen II'
^{133}Cs	p	7/2	0.194	0.052					

Table 3 (revised):Values of $\lambda^2 J(J+1)$ for various isotopes

See D. R. Tovey *et al.* [10] for preferred treatment of this inescapable model-dependency.

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